

Ionospheric Structure, Stochastic TEC, and Scintillation Diagnostics

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1 Extended Abstract

Scintillation diagnostics are one-dimensional time series generated by the cumulative effects of propagation through ionospheric structure. Temporal variation is caused by the motion of the propagation path relative to the three-dimensional ionospheric structure. Cumulative propagation effects map the structure onto a two-dimensional measurement plane. Translation of the propagation path is effectively a scan through the structure in the measurement plane. Interpreting ionospheric diagnostics requires a three-dimensional structure model and a tractable means of calculating statistical measures derived from the diagnostic time series. The most extensively used diagnostic is the spectral density function (SDF) $\Phi^{(1)}(q)$ where q represents the magnitude of a spatial wavenumber $q = 2\pi f_{\text{Dop}}/v_{\text{eff}}$. The effective scan velocity v_{eff} depends on the component of the relative motion of the propagation path projected onto the measurement plane and a correction factor that depends on the direction of the scan relative to the magnetic field.

Let $\Phi_{\Delta N_e}(\kappa_x, \kappa_y, \kappa_z)$ represent the three-dimensional SDF of the ionospheric structure. For a one-dimensional scan along the y -axis direction

$$\Phi^{(1)}(\kappa_y) = \iint \Phi_{\Delta N_e}(\kappa_x, \kappa_y, \kappa_z) \frac{d\kappa_x}{2\pi} \frac{d\kappa_z}{2\pi}. \quad (1)$$

Model calculations for interpreting propagation diagnostics follow from an equivalent phase screen derived from path-integrated structure referred to as total electron content (TEC). If the coordinate system x -axis is aligned with the propagation direction,

$$\Phi_{\Delta \text{TEC}}^{(1)}(\kappa_y) = L \iint \frac{\sin^2(\kappa_x L/2)}{(\kappa_x L/2)^2} \Phi_{\Delta N_e}(\kappa_x, \kappa_y, \kappa_z) \frac{d\kappa_x}{2\pi} \frac{d\kappa_z}{2\pi}. \quad (2)$$

Ionospheric structure models start with an isotropic three-dimensional form with geometric transformations to accommodate structure elongation along the magnetic field direction.

Following the analysis of equatorial plasma bubble (EPB) simulations [1] and a configuration-space model [2] we propose a model that assigns the stochastic structure to two-dimensional planes that cross the magnetic field:

$$\Phi_{\Delta N_e}(\kappa_x, \kappa_y, \kappa_z) = \Phi_{\Delta N_e}(\kappa_s, \kappa_t) 2\pi \delta(\kappa_{\parallel}), \quad (3)$$

where

$$\begin{bmatrix} \cdot \\ \kappa_s \\ \kappa_t \end{bmatrix} = \begin{bmatrix} C_{21}^T & C_{22}^T & C_{23}^T \\ C_{31}^T & C_{32}^T & C_{33}^T \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix} \quad (4)$$

The matrix elements C_{ij} are components of a unitary matrix that rotates the measurement space coordinates into field-aligned coordinates. The T superscript indicates transposition for application to spatial wavenumber components. The singular behavior along the spatial wavenumber corresponding to the magnetic field direction follows the assumption of no variation along the field lines within the structured data volume. The following isotropic cross-field SDF completes the model:

$$\Phi_{\Delta N_e}^{(2)}(\kappa_s, \kappa_t) = C_s \begin{cases} \kappa^{-p_1} & \text{for } \kappa \leq \kappa_0 \\ \kappa_0^{p_2-p_1} \kappa^{-p_2} & \text{for } \kappa > \kappa_0 \end{cases}, \quad (5)$$

where κ is the radial wavenumber. The proposed ionospheric model is defined by four parameters, namely the two-dimensional turbulent strength, C_s , the radial power-law indices p_n and the break-scale wavenumber κ_0 . The corresponding one-dimensional SDF is

$$\Phi_{\Delta N_e}^{(1)}(\kappa_y) = C_p \begin{cases} \kappa_y^{-\eta_1} & \text{for } \kappa_y \leq \kappa_0 \\ \kappa_0^{\eta_2-\eta_1} \kappa_y^{-\eta_2} & \text{for } \kappa_y > \kappa_0 \end{cases}, \quad (6)$$

where $p_n = \eta_n + 1$. The upper frame of Figure 1 shows a color display of structure model (5). The lower frame is a plot of the one dimensional SDF derived by integrating (5) and the one-dimensional form (6), which verifies the spectral index relation and the break scale. The shape of the three-dimensional SDF is independent of the propagation direction. The geometrical dependence of the scan direction is a purely geometrical correction.

With a complete analytic three-dimensional SDF model in hand the ramifications of path integration can be investigated. More importantly, propagation simulations can be used to verify the equivalent phase-screen model and theoretical propagation models based on two-dimensional propagation. As shown in a companion paper, [3] these models have been very effective for both simulations and diagnostic data interpretation. We find that the equivalent phase screen model is very robust. Moreover the path integrated phase, which is a frequency-dependent scaling of (2) is insensitive to the propagation geometry aside from the geometric scale correction. The single exception is strict cross-field propagation, which is formally two-dimensional. Translating field-aligned paths, aside from strong-scatter effects, provide the most direct in situ structure mapping. The TEC SDF and the in situ SDF are identical aside from the path scaling.

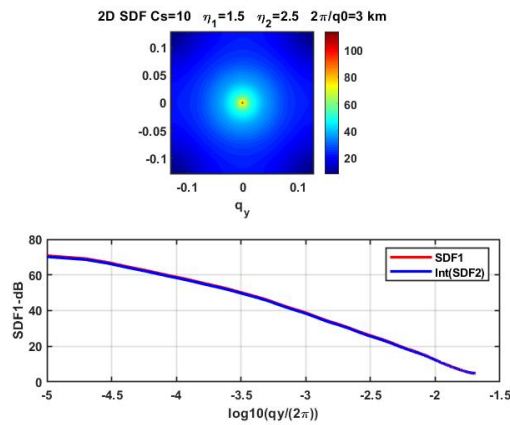


Figure 1. Upper frame shows the isotropic 2D SDF defined by (5). The lower frame compares the one-dimensional SDF defined by (6) with the equivalent result obtained by integrating (5)

References

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